

Threshold ECDSA from ECDSA Assumptions: The Multiparty Case - 2019

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TÜBİTAK - BİLGEM

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2. Oblivious Transfer
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Algorithm 1. $\text{Gen}(1^\kappa)$:

- 1) Uniformly choose a secret key $\text{sk} \leftarrow \mathbb{Z}_q$.
- 2) Calculate the public key as $\text{pk} := \text{sk} \cdot G$.
- 3) Output (pk, sk) .

Algorithm 2. $\text{Sign}(\text{sk} \in \mathbb{Z}_q, m \in \{0, 1\}^*)$:

- 1) Uniformly choose an instance key $k \leftarrow \mathbb{Z}_q$.
- 2) Calculate $(r_x, r_y) = R := k \cdot G$.
- 3) Calculate

$$\text{sig} := \frac{H(m) + \text{sk} \cdot r_x}{k}$$

- 4) Output $\sigma := (\text{sig} \bmod q, r_x \bmod q)$.

Algorithm 3. $\text{Verify}(\text{pk} \in \mathbb{G}, m, \sigma \in (\mathbb{Z}_q, \mathbb{Z}_q))$:

- 1) Parse σ as (sig, r_x) .
- 2) Calculate

$$(r'_x, r'_y) = R' := \frac{H(m) \cdot G + r_x \cdot \text{pk}}{\text{sig}}$$

- 3) Output 1 if and only if $(r'_x \bmod q) = (r_x \bmod q)$.

ECDSA sign is

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The i 'th party knows v_i, w_i and no other party knows these values.

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Remark

Nobody can learn the secrets sk and k in this process, only the signature is created.

OT functionality

Sender

Input: (m_0, m_1)

Output: nothing, $\perp$

(m_0, m_1)
→

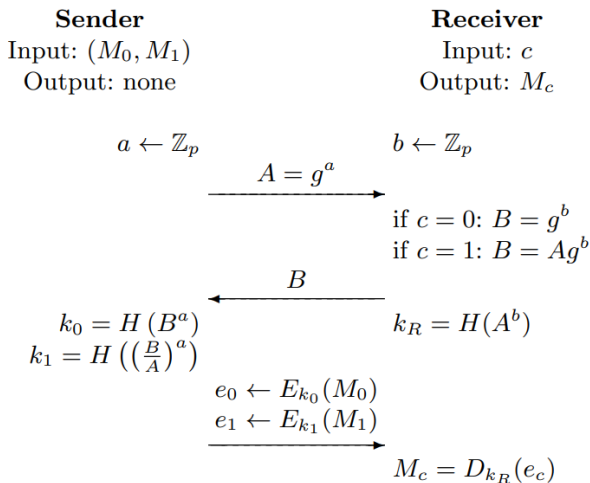
Receiver

Input: $b \in \{0, 1\}$

Output: m_b

Does not know: m_{1-b}

The Simplest Protocol for OT - *Tung Chou and Claudio Orlandi*



Protokol:

Bob

b

$$s_0, s_1, \dots, s_{p-1} \leftarrow_{\$} \mathcal{R}$$

$$\begin{aligned} t_0^0 &= s_0, & t_0^1 &= b + s_0 \\ t_1^0 &= s_1, & t_1^1 &= 2^1 b + s_1 \\ t_2^0 &= s_2, & t_2^1 &= 2^2 b + s_2 \\ &\vdots & &\vdots \\ t_i^0 &= s_i, & t_i^1 &= 2^i b + s_i \end{aligned}$$

$$t_i^0 = s_i, \quad t_i^1 = 2^i b + s_i$$

$$y = \sum_{i=0}^{p-1} s_i$$

Alice

$$a = (a_{p-1}, \dots, a_0)_2$$

$$\begin{aligned} &\text{choose } t_0^{a_0} \\ &\text{choose } t_1^{a_1} \\ &\text{choose } t_2^{a_2} \\ &\vdots \\ &\text{choose } t_i^{a_i} \end{aligned}$$

$$x = \sum_{i=0}^{p-1} t_i^{a_i}$$

İspat:

$$\begin{aligned} x + y &= \sum_{i=0}^{p-1} t_i^{a_i} - \sum_{i=0}^{p-1} s_i \\ &\equiv \sum_{i=0}^{p-1} (a_i \cdot 2^i b + s_i) - \sum_{i=0}^{p-1} s_i \\ &\equiv b \sum_{i=0}^{p-1} a_i \cdot 2^i \\ &\equiv ab \end{aligned}$$

Sender

Two messages
 α^0, α^1

$$b \leftarrow \mathbb{Z}_q$$

$$B := b \cdot G$$

$$r^0 := H(b \cdot A)$$

$$r^1 := H(b \cdot (A - B))$$

Compute a challenge:
 $\varepsilon := H(H(r^0)) \oplus H(H(r^1))$ Abort if
 $r^1 \neq H(H(r^0))$

$$\tilde{\alpha}^0 := \alpha^0 + r^0$$

$$\tilde{\alpha}^1 := \alpha^1 + r^1$$

 $\xrightarrow{B}$ $\xleftarrow{A}$ $\xrightarrow{\varepsilon}$ $\xleftarrow{r^1}$ $\xrightarrow{H(r^0), H(r^1)}$ $\xrightarrow{\tilde{\alpha}^0, \tilde{\alpha}^1}$

Receiver

Choice bit
 $w \in \{0, 1\}$

DL-ZKProve(B)

$$a \leftarrow \mathbb{Z}_q$$

$$A := a \cdot G + w \cdot B$$

$$r^w := H(a \cdot B)$$

Compute a response
 $r^1 := H(H(r^w)) \oplus (w \cdot \varepsilon)$ Abort if
 $H(r^w)$, Does not match the
one calculated itselfAbort if
 $\varepsilon \neq H(H(r^0)) \oplus H(H(r^1))$

$$\alpha^w = \tilde{\alpha}^w - r^w$$

Public Key , Pad Transfer , Verification , Message Transfer

Corallated OT - $\mathcal{F}_{\text{COTe}}^l$ fonksiyonu

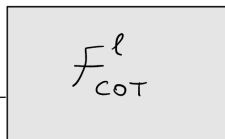
Sender

correlation:

$$\alpha \in G_1 \times G_2 \times \dots \times G_\ell$$

$$t_S \leftarrow_{\$} G_1 \times G_2 \times \dots \times G_\ell$$

Output t_S



Receiver

$w \in \{0,1\}^\ell$; choice

if $w_i = 0 \rightarrow -t_{S_i}$

if $w_i = 1 \rightarrow \alpha_i - t_{S_i}$

which means:

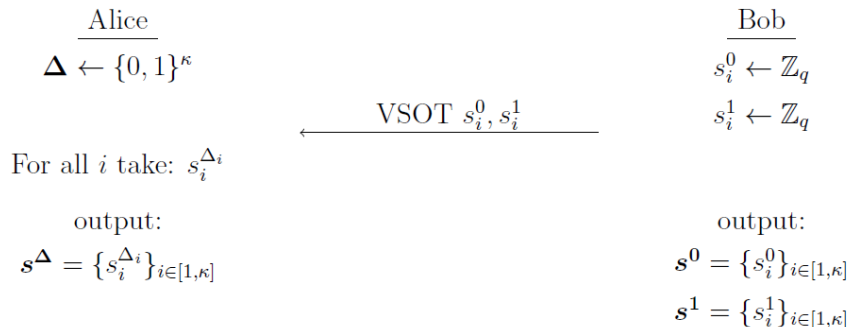
$$t_R : \{w_i \cdot \alpha_i - t_{S_i}\}_{i \in [1,\ell]}$$

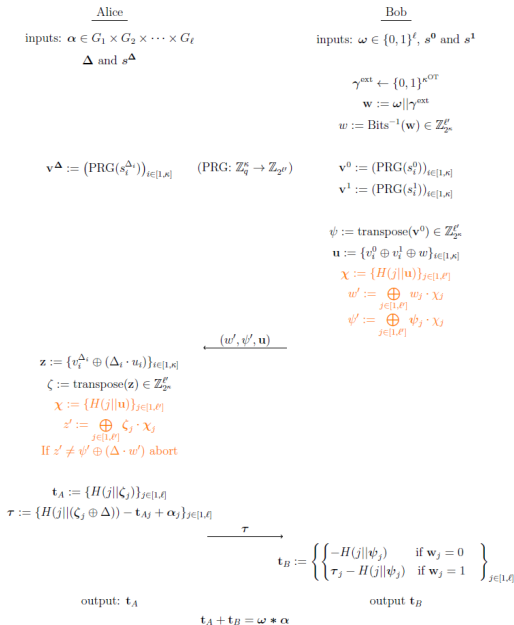
Output t_R

$$t_S + t_R = w * \alpha$$

KOS (Keller, Orsini, Scholl) Setup

- KOS Setup ve KOS Extension protokolleri yukarıdaki $\mathcal{F}_{\text{COTe}}^\ell$ fonksiyonunu gerçekleştirir.





Why Consistency Check works?

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$$z_i = v_i^{\Delta_i} \oplus \left[\Delta_i \cdot (v_i^0 \oplus v_i^1 \oplus w) \right]$$

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Case 1: $\Delta_i = 0$

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Case 2: $\Delta_i = 1$

$$z_i = v_i^1 \oplus (v_i^0 \oplus v_i^1 \oplus w) = v_i^0 \oplus w$$

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Which means:

$$z_i = \begin{cases} v_i^0 & \text{if } \Delta_i = 0 \\ v_i^0 \oplus w & \text{if } \Delta_i = 1 \end{cases}$$

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So we can write $z_i = v_i^0 \oplus (\Delta_i \cdot w)$

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Alice calculates:

$$t_A := \{H(j \parallel \zeta_j)\}_{j \in [1, \ell']}$$

$$\tau := \{H(j \parallel (\zeta_j \oplus \Delta)) - t_{A_j} + \alpha_j\}_{j \in [1, \ell']}$$

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Summing two shares:

$$\begin{aligned}t_{Aj} + t_{Bj} &= H(j\|\psi_j) - H(j\|\psi_j) \\ &= 0 \\ &= \alpha_j \cdot w_j \quad (\text{since } w_j = 0)\end{aligned}$$

Case 2: $w_j = 1$

$$\zeta_j = \psi_j \oplus \Delta$$

$$t_{Aj} = H(j \| (\psi_j \oplus \Delta))$$

$$\tau_j = H(j \| \psi_j) - t_{Aj} + \alpha_j$$

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Case 2: $w_j = 1$

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Calculation of Bob::

$$\begin{aligned}t_{Bj} &= \tau_j - H(j \| \psi_j) \\ &= (H(j \| \psi_j) - t_{Aj} + \alpha_j) - H(j \| \psi_j) \\ &= -t_{Aj} + \alpha_j\end{aligned}$$

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Summing t_A and t_B , we have:

$$\begin{aligned}t_{Aj} + t_{Bj} &= t_{Aj} - t_{Aj} + \alpha_j \\ &= \alpha_j \\ &= \alpha_j \cdot w_j \quad (\text{since } w_j = 1)\end{aligned}$$

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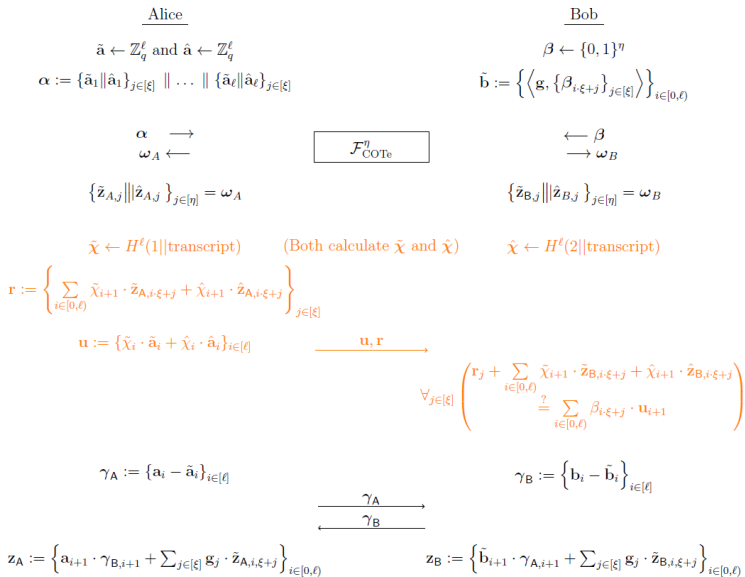
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In both cases

$$t_{Aj} + t_{Bj} = \alpha_j \cdot w_j$$

$\pi_{2\text{PMul}}^\ell$ - Two-party Multiplication



$$\mathbf{z}_A + \mathbf{z}_B = \mathbf{a} \cdot \mathbf{b}$$

$$z_A = \left\{ a_i \cdot \gamma_{B,i} + \sum_j g_j \cdot \tilde{z}_{A,i-\xi+j} \right\}_{i \in [\ell]} \quad z_B = \left\{ \tilde{b}_i \cdot \gamma_{A,i} + \sum_j g_j \cdot \tilde{z}_{B,i-\xi+j} \right\}_{i \in [\ell]}$$

Why MtA Works?

$$z_A = \left\{ a_i \cdot \gamma_{B,i} + \sum_j g_j \cdot \tilde{z}_{A,i-\xi+j} \right\}_{i \in [\ell]} \quad z_B = \left\{ \tilde{b}_i \cdot \gamma_{A,i} + \sum_j g_j \cdot \tilde{z}_{B,i-\xi+j} \right\}_{i \in [\ell]}$$

Summing these values,

$$z_A + z_B = a \cdot \gamma_B + \tilde{b} \cdot \gamma_A + \sum_j g_j \cdot (\tilde{z}_A + \tilde{z}_B)_{i-\xi+j}$$

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Hence,

$$z_A + z_B = a \cdot b - \tilde{a} \cdot \tilde{b} + \tilde{a} \cdot \tilde{b} = a \cdot b$$

Why Consistency Check Works?

Alice calculates:

$$r = \sum_i \tilde{x}_i \cdot \tilde{z}_{A,i} + \hat{x}_i \cdot \hat{z}_{A,i}, \quad u = \tilde{x} \cdot \tilde{a} + \hat{x} \cdot \hat{a}$$

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Bob checks that:

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- Each party P_i for $i \in P$ samples $k_i \leftarrow \mathbb{Z}_q$ and a pad $\phi_i \leftarrow \mathbb{Z}_q$, set these values as $\psi_i^1 := \{k_i, \frac{\phi_i}{k_i}\}$ and commit the pad ϕ_i

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- For $\rho \in [\log_2 t]$:

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- For $\rho \in [\log_2 t]$:
 - For each pair of parties P_i, P_j in each contiguous non-overlapping subgroup of 2^ρ parties from $\mathcal{P}$, if P_i and P_j have not previously interacted during the course of this invocation of $\pi_{\text{inv}}^{n,t}$, then they send

$$(\psi_i^\rho) \quad \text{and} \quad (\psi_j^\rho)$$

to $\mathcal{F}_{2\text{PMul}}^\ell$, respectively, and receive

$$(\zeta_i^{\rho,j}) \quad \text{and} \quad (\zeta_j^{\rho,i}).$$

t-party Modular Inverse Sampling

- Each party P_i for $i \in P$ samples $k_i \leftarrow \mathbb{Z}_q$ and a pad $\phi_i \leftarrow \mathbb{Z}_q$, set these values as $\psi_i^1 := \{k_i, \frac{\phi_i}{k_i}\}$ and commit the pad ϕ_i
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 - For each pair of parties P_i, P_j in each contiguous non-overlapping subgroup of 2^ρ parties from $\mathcal{P}$, if P_i and P_j have not previously interacted during the course of this invocation of $\pi_{\text{inv}}^{n,t}$, then they send

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to $\mathcal{F}_{2^{\text{PMul}}}^\ell$, respectively, and receive

$$(\zeta_i^{\rho,j}) \quad \text{and} \quad (\zeta_j^{\rho,i}).$$

- Each party P_i privately computes $\psi_i^{\rho+1}$ to be the element-wise sum of its output shares for round ρ :

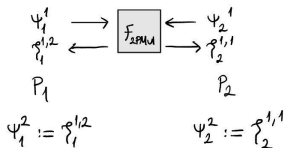
$$\psi_i^{\rho+1} := \left\{ \sum_{j \in P^{\rho,i}} \zeta_{i,\ell}^{\rho,j} \right\}_{\ell \in [2]}$$

where $P^{\rho,i} \subseteq \mathcal{P}$ is the subgroup with whom P_i interacted in round ρ such that $|P^{\rho,i}| = 2^{\rho-1}$.

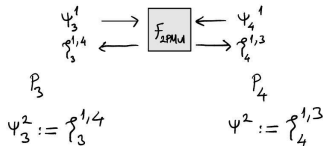
Step-2 (4 Party Example)

$p=1$: All parties come together two by two.

(P_1, P_2)

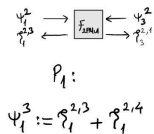


(P_3, P_4)

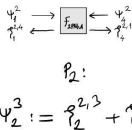


$p=2$: All parties come together four by four.
 (P_1, P_2, P_3, P_4)

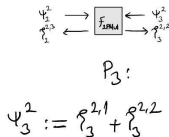
(P_1, P_3)



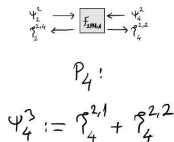
(P_1, P_4)



(P_2, P_3)



(P_2, P_4)



$$\psi_1^1 \cdot \psi_2^1 \cdot \psi_3^1 \cdot \psi_4^1$$

$$(\psi_1^2 + \psi_2^2) \cdot (\psi_3^2 + \psi_4^2)$$

$$\psi_1^2 \cdot \psi_3^2 + \psi_1^2 \cdot \psi_4^2 + \psi_2^2 \cdot \psi_3^2 + \psi_2^2 \cdot \psi_4^2$$

$$\psi_1^{2,3} + \psi_3^{2,1} + \psi_1^{2,4} + \psi_4^{2,1} + \psi_2^{2,3} + \psi_3^{2,2} + \psi_2^{2,4} + \psi_4^{2,2}$$

$$\psi_1^3 + \psi_3^3 + \psi_2^3 + \psi_4^3$$

10 Party Example

2-by-2 group

(P_1, P_2)

(P_3, P_4)

(P_5, P_6)

(P_7, P_8)

(P_9, P_{10})

4-by-4 group

(P_1, P_2, P_3, P_4)

(P_5, P_6, P_7, P_8)

(P_9, P_{10})

(P_1, P_3) (P_1, P_4)

(P_5, P_7) (P_5, P_8)

(P_2, P_3) (P_2, P_4)

(P_6, P_7) (P_6, P_8)

8-by-8 group

$(P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8)$

(P_9, P_{10})

(P_1, P_5) , (P_1, P_6) , (P_1, P_7) , (P_1, P_8)

(P_2, P_5) , (P_2, P_6) , (P_2, P_7) , (P_2, P_8)

(P_3, P_5) , (P_3, P_6) , (P_3, P_7) , (P_3, P_8)

(P_4, P_5) , (P_4, P_6) , (P_4, P_7) , (P_4, P_8)

10-by-10

$(P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10})$

(P_1, P_9) , (P_1, P_{10})

(P_2, P_9) , (P_2, P_{10})

(P_3, P_9) , (P_3, P_{10})

(P_4, P_9) , (P_4, P_{10})

(P_5, P_9) , (P_5, P_{10})

(P_6, P_9) , (P_6, P_{10})

(P_7, P_9) , (P_7, P_{10})

(P_8, P_9) , (P_8, P_{10})

- Each party P_i sets $\{u_i, \tilde{v}_i\} = \psi_i^{\lceil \log_2(t) \rceil + 1}$ and so

$$\sum_{i \in \mathcal{P}} u_i = \prod_{i \in \mathcal{P}} k_i \quad \text{and} \quad \sum_{i \in \mathcal{P}} \tilde{v}_i = \prod_{i \in \mathcal{P}} \frac{\phi_i}{k_i}.$$

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t-party Modular Inverse Sampling

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- Each party calculates

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and aborts if the pad is equal to zero, or if

$$\sum_{j \in \mathcal{P}} \Gamma_j^1 \neq \phi \cdot G.$$

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and aborts if the pad is equal to zero, or if

$$\sum_{j \in \mathcal{P}} \Gamma_j^1 \neq \phi \cdot G.$$

- Each party P_i computes $v_i := \tilde{v}_i / \phi$ and takes (u_i, v_i, R) as its output.

$$\begin{array}{ccc}
\underline{P_i} & & \underline{P_j} \\
p_i(x) \leftarrow \text{poly-degre}(t-1) & & \\
p_i(j) & \xrightarrow{\quad p_i(j) \quad} & \\
p(i) = \sum_{j \in P} p_j(i) & \xleftarrow{\quad p_j(i) \quad} & \\
T_i := p(i) \cdot G & & \\
c_i := \text{commit}(T_i) & \xrightarrow{\quad c_i \quad} & \\
& \xleftarrow{\quad c_j \quad} & \\
& \xrightarrow{\quad T_i \quad} & \\
\text{open}(c_j) \stackrel{?}{=} T_j & \xleftarrow{\quad T_j \quad} & \\
\pi_i := \text{DLZKP}(p(i)) & \xrightarrow{\quad \pi_i \quad} & \\
\text{ZKVerify}(\pi_j) \stackrel{?}{=} 1 & \xleftarrow{\quad \pi_j \quad} & \\
\forall x \in [1, n-t-1] & & \\
J^x = [x, x+t] \text{ and } J^{x+1} = [x+1, x+t+1] & & \\
\sum_{j \in J^x} \lambda_j^{J^x}(0) \cdot T_j \stackrel{?}{=} \sum_{j \in J^{x+1}} \lambda_j^{J^{x+1}}(0) \cdot T_j & &
\end{array}$$

$$\underline{P_i}$$

$$\text{Call } (u_i, v_i, R) \leftarrow \mathcal{F}_{\text{inv}}^{n,t}$$

$$sk_i := \lambda_i^P(0) \cdot p(i)$$

$$\begin{array}{l} \{sk_i, v_i\} \longrightarrow \\ \{w_i^{j,1}, w_i^{j,2}\} \longleftarrow \end{array}$$

$$\underline{P_j}$$

$$\boxed{\mathcal{F}_{2\text{PMul}}^2}$$

$$\begin{array}{l} \longleftarrow \{v_j, sk_j\} \\ \longrightarrow \{w_j^{i,1}, w_j^{i,2}\} \end{array}$$

$$w_i^{j,1} + w_j^{i,1} = sk_i \cdot v_j$$

$$w_i^{j,2} + w_j^{i,2} = sk_j \cdot v_i$$

$$w_i := sk_i \cdot v_i + \sum_{j \in P/\{i\}} w_i^{j,1} + w_i^{j,2}$$

$$\Gamma_i^2 := v_i \cdot pk - w_i \cdot G$$

$$\Gamma_i^3 := w_i \cdot R$$

$$c_i := \text{commit}(\Gamma_i^2, \Gamma_i^3)$$

$$\xrightarrow{c_i}$$

$$\xleftarrow{c_j}$$

$$\xrightarrow{\Gamma_i^2, \Gamma_i^3}$$

$$\xleftarrow{\Gamma_j^2, \Gamma_j^3}$$

$$\text{open}(c_j) \stackrel{?}{=} (\Gamma_j^2, \Gamma_j^3)$$

$$\sum_{j \in P} \Gamma_j^2 \stackrel{?}{=} 0$$

$$\sum_{j \in P} \Gamma_j^3 \stackrel{?}{=} pk$$

$$\text{sig}_i := H(m) \cdot v_i + w_i \cdot r_x$$

$$\text{sig} = \sum_{i \in P} \text{sig}_i$$

- Setup: 5 rounds (key generation) + 3 rounds (“auxiliary setup”)
- Signing: $6 + \lceil \log t \rceil$ rounds (only the last is online)

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Protocol	Signing		Setup	
	$t = 2$	$t = 20$	$n = 2$	$n = 20$
This Work	8.6	31.5	48.5	242
[GG18]	77	509	–	–
[LNR18]	304	5194	~11000	~28000
[BGG17]	~650	~1500	–	–
[GGN16]	205	1136	–	–
[Lin17]	36.8	–	2435	–
[DKLs18]	3.8	–	43.4	177

Table 4: **Wall-clock Time Comparison to Other Works.** Note that benchmarking environments are non-identical, but all benchmarks are networkless or over LAN.